Physics II
ISI B.Math
Final Exam : Nov 10, 2014

Total Marks: 50

Answer all questions

1. **(Marks :** $2 \times 5 = 10$**)**
Choose the correct option. No explanation is necessary.

(a) A system is in thermal equilibrium with a reservoir at a given temperature T. The system can be in any one of its microstates compatible with the condition that the total energy of the system plus reservoir is fixed. Which of the following statements is true ?

(i) All possible microstates of the system are equally probable.
(ii) Microstates of the system with high energy are less probable than those with low energy.
(iii) Microstates of the system with high energy are more probable than those with low energy.

(b) Which of the following processes result in an increase of entropy of the universe ?

(i) An insulated chamber has a partition in the middle. The left half contains an ideal gas and the right half is empty. The partition is removed.
(ii) An insulated chamber has a partition in the middle. The left half contains an ideal gas. The right half contains the same amount of the same ideal gas at the same temperature and pressure as the left half. The partition is removed.
(iii) A cylinder fitted with a frictionless piston contains 1 litre of an ideal gas at room temperature. The gas is allowed to expand to 2 litres by very slowly pulling up the piston while maintaining the gas at room temperature.

(c) Unpolarized sunlight is incident at an angle of $56°$ (Brewster's angle) on a slab of glass. The transmitted wave will be

(i) absent
(ii) plane polarized with the electric field vector perpendicular to the plane of incidence.
(iii) plane polarized with the electric field vector parallel to the plane of incidence.
(iv) partially polarized.

(d) In a double slit arrangement, the wavelength λ of the light source is 405 nm, the slit separation is 20 μm and the slit width is 4μm . The number of bright interference fringes within the central peak of the diffraction envelope is

(i) 11
(ii)10
(iii) 9

(e) If the source of light used in a Young's double slit is changed from red to violet

(i) the fringes will become brighter
(ii) the central bright fringe will become a dark fringe
(iii) Consecutive fringes will come closer.

2. **(Marks : 3 + 4 + 3 = 10)**

(a) A thermally insulated cylinder closed at both ends is fitted with a heat conducting, frictionless piston which divides the cylinder into two equal parts containing the same amount of ideal gas with constant C_V at temperatures T_1 and T_2 and pressures P_1 and P_2 respectively. The piston is released and the system reaches equilibrium. Find
(i) the final temperature and pressure
(ii) the change in entropy of the system and show that it is ≥ 0.

(b) Plot the $T - S$ diagram of a Carnot cycle. Derive an expression for the efficiency of the Carnot engine in terms of T_h (the temperature of the heat source) and T_l (the temperature of the heat sink) directly from the $T - S$ diagram.

3. **(Marks : 3 + 2 + 3 + 2 = 10)**

The energy levels of a quantum mechanical oscillator with frequency ν are given by $\epsilon_n = (n + \frac{1}{2})h\nu$, where $n = 0, 1, 2 \cdots$ and h is Planck's constant. Consider a collection of $3N$ independent quantum mechanical oscillators in equilibrium with a heat reservoir at temperature T.

(a) Find an expression for the partition function Z of the system as a function of T.
(b) Find the Helmholtz free energy F for this system.
(c) Find the average energy $\bar{E}$ for the system.
(d) Find the limiting forms of the average energy for $T \gg \frac{h\nu}{k}$ and $T \ll \frac{h\nu}{k}$.

4. **(Marks : 4 + 2 + 4 = 10)**

In a double slit arrangement monochromatic light of wavelength λ from a distant source is incident upon two slits, each of width w ($w \ll \lambda$) and an interference pattern is seen on a distant screen. A thin piece of mica of thickness δ and index of refraction μ is placed between one of the slits and the screen, and the intensity of the central point C on the screen (which is equidistant from both the slits) is measured as a function of the thickness δ. If the intensity for $\delta = 0$ is given by I_0,

(a) What is the intensity at C as a function of δ?
(b) For what values of δ is the intensity at C a minimum ?
(c) Suppose that the width of one of the slits is now increased to $2w$, the other width remaining the same. What is the intensity at point C as a function of δ? Assume that the mica does not absorb any light.

5. **(Marks : 4 + 2 + 4 = 10)**

(a) A glass surface is coated by an oil film of uniform thickness 1.00×10^{-4} cm. The index of refraction of the oil is 1.25 and that of glass is 1.50. Find the wavelengths of light in the visible region (400nm $--$750nm) which are strongly transmitted by the oil film under normal incidence.

(b) Discuss the state of polarization when the x and y components of the electric field are given by $E_x = E_0 \sin(\omega t + kz), E_y = E_0 \cos(\omega t + kz)$.

(c) The intensity distribution on the screen due to the light of wavelength λ transmitted through a grating of N slits of width b and slit separation d is $I = I_0 \left(\frac{\sin^2 \beta}{\beta^2}\right) \frac{\sin^2 N\gamma}{\sin^2 \gamma}$ where $\beta = \frac{\pi b \sin \theta}{\lambda}$ and $\gamma = \frac{\pi d \sin \theta}{\lambda}$. Assuming that the slit widths are small ($b \ll \lambda$), find the locations of the principal maxima and the minima. Make a rough plot of the interference factor $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$ vs γ for $N = 5$.